2022A12 DISTANCE VS. DISPLACEMENT

Level 1: To find the total distance traveled by the trailers carrying the wind turbine blade, we simply add all of the distances stated in the word problem.

$$Distance = 2 + 10 + 4 + 3 + 2 + 15 + 5 + 6 + 1 = \mathbf{48\ miles}$$

To find the displacement of the trailers, we have to take into account the direction of each distance stated in the word problem. It is helpful if we think of it in terms of an x/y coordinate plane.

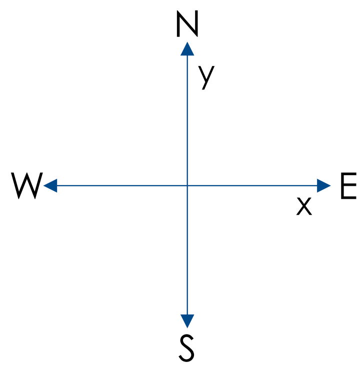

Figure 1: X/Y coordinate plane with cardinal directions

Now we can add together distances which are along the same axis. Let's start with the x-axis, where East is positive, and West is negative.

$$Displacement(x) = 2 + 4 - 2 - 5 - 1 = \mathbf{-2\ miles}$$

The displacement of the trailers along the x-axis is -2 miles, or 2 miles to the West.

Next, we can move to the y-axis, where North is positive, and South is Negative.

$$Displacement(y) = 10 + 3 + 15 - 6 = \mathbf{22\ miles}$$

The displacement of the trailers along the y-axis is 22 miles, or 22 miles to the North.

Using the Pythagorean Theorem, we can find the magnitude of the displacement from the laydown yard to the project site.

$$Displacement = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + 22^2} = \mathbf{22.1\ miles}$$

Level 2: To find the total distance traveled, we once again add the distances stated in the word problem.

$$Distance = 8 + 3 + 5 + 2 + 7 + 10 = \mathbf{35\ miles}$$

To find the displacement, however, we need to breakdown each distance into its x and y components. To do this, we can use sine and cosine functions. Remember, sine relates to the y-component and cosine relates to the x-component.

This part is a little tricky to visualize. Our directions are given as compass degrees, but to solve for the x and y components using trigonometric functions, we can't just use the compass degree value.

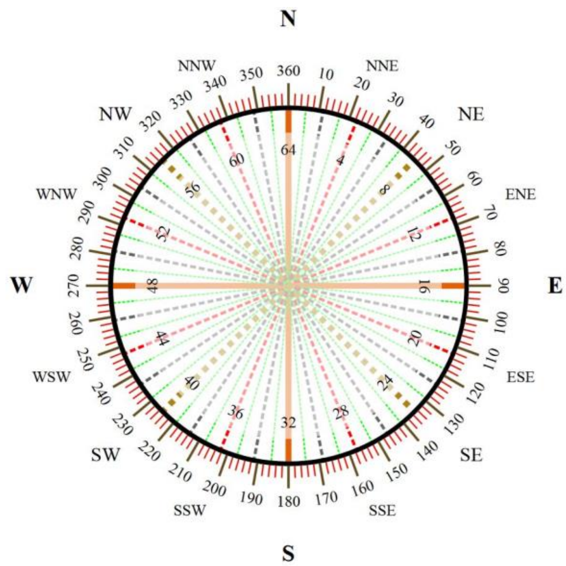

Figure 2: Degree compass

The best way to think about this is in terms of the direction's relation to x-axis. Let's take a look at the first distance, 8 miles at 230° Southwest, as an example.

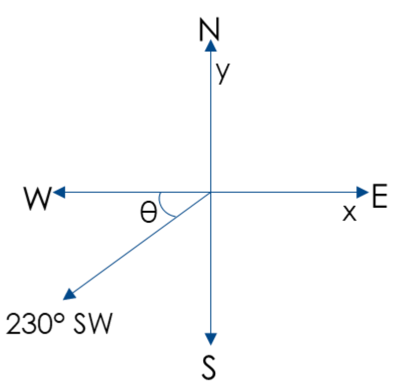

Figure 3: First distance on x/y coordinate plane

If we refer to the degree compass above, we can see that West (x-axis) is at 270°. By subtracting our distance direction (230° SW) from 270°, we get an angle (θ) of 40° with the x-axis. Now we can use sine and cosine to solve for the x and y-components of our first distance.

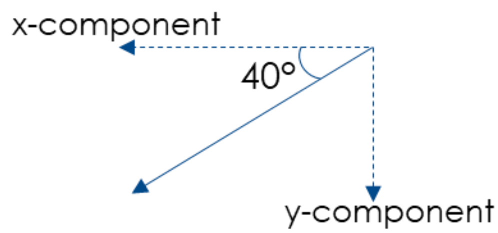

Figure 4: X/Y components

$$x - Component = distance \cdot \cos(\theta) = 8 \cdot \cos(40°) = \mathbf{6.1\ miles}$$

$$y - Component = distance \cdot \sin(\theta) = 8 \cdot \sin(40°) = \mathbf{5.1\ miles}$$

It is important to remember the orientation of our distance. At 230° Southwest, the x-component is 6.1 miles West and the y-component is 5.1 miles South. So the x-component would be -6.1 miles and the y-component would be -5.1 miles.

Using the same strategy for the rest of the distances and directions given in the word problem, we can solve for all the x and y components. Below is a table of the distances and their components.

DISTANCES	X-COMPONENT	Y-COMPONENT
8 miles at 230° Southwest	-6.1	-5.1
3 miles at 140° Southeast	1.9	-2.3
5 miles at 260° West-southwest	-4.9	-0.9
2 miles at 170° South-southeast	0.3	-2.0
7 miles at 320° Northwest	-4.5	5.4
10 miles at 250° West-southwest	-9.4	-3.4

Table 1: X/Y component solutions for each distance given in the Level 2 word problem

Now that we have x and y components for each distance, we can sum the components in the x and y directions like we did in the level 1 problem.

$$Total\ x - component = -6.1 + 1.9 - 4.9 + 0.3 - 4.5 - 9.4 = \mathbf{-22.7\ miles}$$

$$Total\ y - component = -5.1 - 2.3 - 0.9 - 2.0 + 5.4 - 3.4 = \mathbf{-8.3\ miles}$$

The total displacement of the trailers in the x-direction was -22.7 miles, or 22.7 miles West. The total displacement in the y-direction was -8.3 miles, or 8.3 miles South.

Once again, we can use the Pythagorean Theorem to solve for the magnitude of the displacement from the laydown yard to the project site.

$$Displacement = \sqrt{x^2 + y^2} = \sqrt{(-22.7)^2 + (-8.3)^2} = \mathbf{24.2\ miles}$$