2023A1 FUNCTIONS, AREA

Level 1:

For this first question, we want to set the width of the crane pad as our unknown variable, X. This way, we can set the length of our crane pad equal to 1.5X. Then we can create an equation to solve for X using the given perimeter.

$$2 * Length + 2 * Width = Perimeter$$

$$2 * (1.5X) + 2 * (X) = 250\ feet$$

$$3X + 2X = 250\ feet$$

$$5X = 250\ feet$$

$$X = 50\ feet$$

The width of the crane pad is equal to 50 feet and the length of the crane pad is equal to 1.5 times 50 feet, or 75 feet. The area of the crane pad will be the length times the width, which comes out to 3750 feet^2.

Level 2:

For the second question, we can once again set the width of the crane pad as our unknown variable, X. This time, however, our relationship between the length and width is a little more complicated. Our relationship between length and width will look like this:

$$Length = 2X - 45.$$

We also know that the area of this crane pad is the same as the Level 1 question. So we can set up an equation using the given area to solve for X.

$$Length * Width = Area$$

$$(2X - 45) * X = 3750\ feet^2$$

$$2X^2 - 45X = 3750 feet^2$$

$$2X^2 - 45X - 3750 = 0$$

Now we have a solvable equation for X. We can use the quadratic equation to solve for X.

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, for\ the\ form\ aX^2 - bX - c = 0$$

$$X = \frac{45 \pm \sqrt{(-45)^2 - 4(2)(-3750)}}{2(2)}$$

$$X = \frac{45 \pm \sqrt{2025 + 30000}}{4}$$

$$X = \frac{45 \pm \sqrt{32025}}{4}$$

This is a bit of a complicated radical we have to deal with here, but it can be simplified a little further by hand… or just plugged into a calculator.

We know that 32,025 is divisible by 5 and is likely also divisible by 25. Dividing 32035 by 25 gives 1281.

$$X = \frac{45 + 5\sqrt{1281}}{4}, \frac{45 - 5\sqrt{1281}}{4}$$

This is as simple as the solution will go, we now need a calculator to solve. The two solutions we obtain are 55.98 and -33.49. Since we cannot have a negative width, we know that 55.98 feet will be our answer, and we can round to 56 feet for simplicity.

Now that we know our width, we can solve for our crane pad length.

$$Length = 2 * 56 - 45 = 67\ feet$$

The length and width of this crane pad is 56 feet by 67 feet. We can use a graphing calculator or other graphing software to check our answer.

We plugged our equation ($2X^2 - 45X - 3750 = 0$) into the Desmos graphing software, which is online and free to use. The result is below.

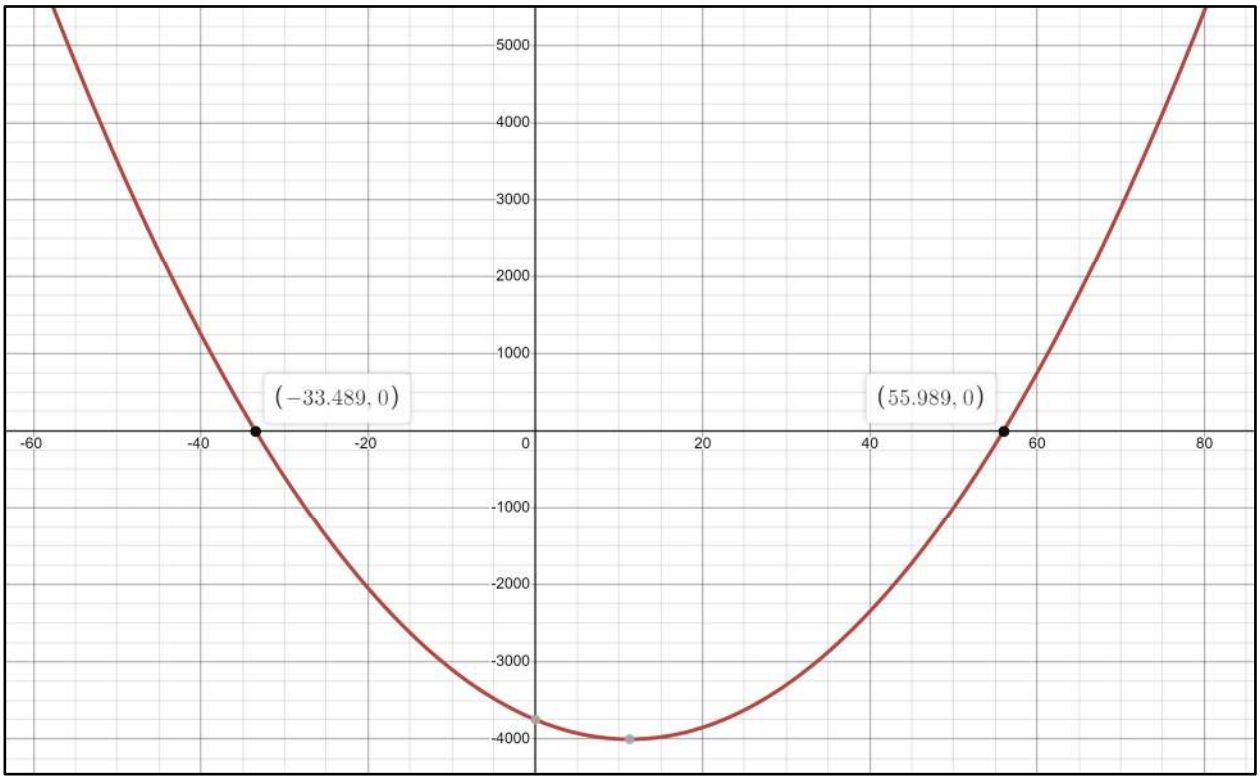

Figure 1: Solution from Desmos calculator

As we can see, our math was correct, the solutions to our equation were about 56 and about -33.5.